Final Dynamics Project: Triple Rigid Body Pendulum

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***Abstract*—We developed a model for the motion of a rigid body triple pendulum in MATLAB. We used Lagrangian methods to derive the equations of motion for each of the rigid bodies. The model includes a series of 3 coupled oscillators, and given an initial position and/or an initial velocity to any of the masses, we can calculate the motion of the triple pendulum. Using MATLAB, we were able to evaluate the position of the pendulum through the numberical integrator, ODE45. We compared our model to experimental data taken from an actual triple pendulum, and found that they agreed very well with the periodic patterns.**

I. Background

In preparation for modeling the triple pendulum, we reviewed the related notes and lectures from Professor Christopher Lee regarding the equations of motion, energy equations, as well as Lagrangian Methods. When modeling the triple pendulum, we consider the movement to be 2 Dimensional, rotating about the z-axis. Lagrangian equations are based off the energy of the system, thus showing whether or not energy is conserved within the system (in which our case, energy is conser ed).

For the experiement, we continued to use the same double pendulum as our past experiment with an additional link. Using motion tracking software, we were able to retriethe posdata (Thanks to Nick Eyre and Jeff Holzgrafe) in terms of time, position, and velocity. We import this data into MATLAB and compare it to our simulated data.

Fig. 1. CAD Model of the Triple Pendulum

II. Learning Objectives

This entire semester, we have been working with pendulum models. We have derived the equations of motion previously using forces in cartesian coordinates and each time wound up with 5-10 pages of derivations. We wanted to use this project to teach ourselves Lagrange's equations, which are specifically useful applied to pendulum systems and can simply the algebra significantly. Having successfully modelled a rigid body double pendulum, we decided to take on the challege of modellling a rigid body triple pendulum. We wanted to further study chaotic behavior through comparison of the simulation and the experiment. Specifically, under what initial conditions is the model more prone to chaotic behavior? Under what conditions does it behave periodically? Given this, in which situations does our simulation match the experiment for an extended period of time?

III. System Model

When modeling a triple pendulum, a clear choice for the generalized coordinates are the angles, $\theta_1\theta_2\theta_3$, since x and y in cartesian coordinates are coupled to each other.

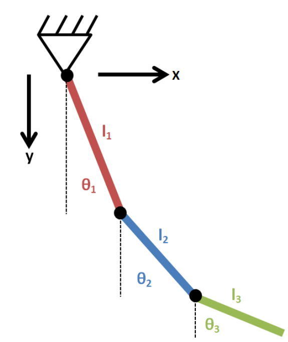

Fig. 2. Diagram of our system

Our system includes the moments of inertia for each of the three rigid bodies, assuming uniform density across the bodies. The values for the moments of inertia come from the values in the CAD model of the actual pendulum. We did not include damping forces in our model. All the initial conditions (position, velocity) and system parameters (mass, length) for each of the links in the simulation were taken from the experimental data and CAD model, in order to make the models match as closely as possible.

To derive the equations of motion using Lagrange's equations, we first calculated kinetic energy (T) and potential energy (U). Our state variables $(x_i, \dot{x}_i, y_i, \dot{y}_i)$ define the position of the center of mass of each of the rigid bodies in cartesian coordinates.

$$T = \sum \frac{1}{2} m_i v_i^2 + \frac{1}{2}[I]_i \dot{\theta_i}^2$$

$$U = m_i g y_i$$

where $v_i = \dot{x_i}^2 + \dot{y_i}^2$ and $I_i = I_z z_i$, due to the pendulum's rotation about the z-axis.

Lagrangian Method takes the difference of kinetic energy and potential energy, $\mathcal{L} = T - U$.

IV. Results

A. Conservation of Energy

As a prelimiary check to validate our model, we plotted the energies over the duration of the simulation. The Total Energy is conserved, as shown in figure 3

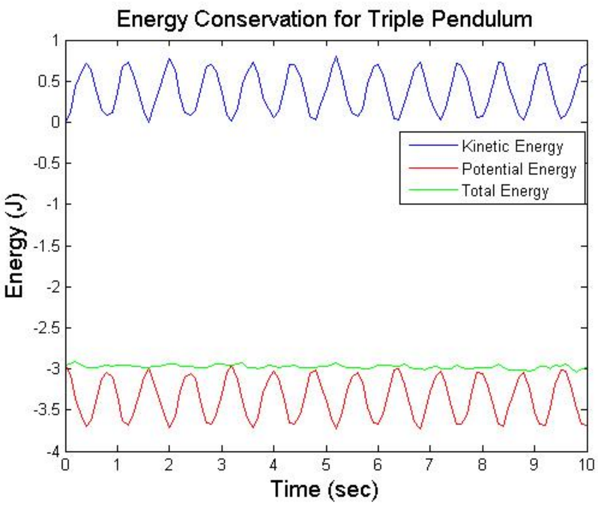

Fig. 3. Energy is conserved

B. Path of Links

Figure 4 shows the range of motion of each of the links over the time span of the simulation. As is expected, the motion of Link 1 is periodic, while the motion of links 2 and 3 have wider ranges of motion because of the added degrees of freedom.

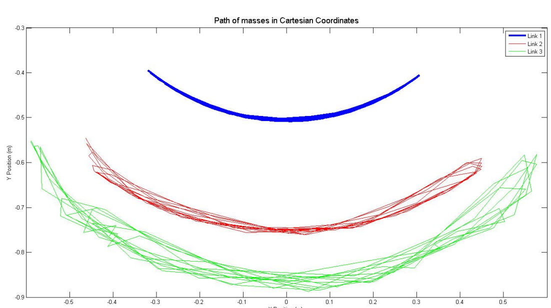

Fig. 4. Link 3 has the widest range of motion

C. Position over Time

The positions of the third link have the higest fluctuation, due to its 3 degrees of freedom. The experimental and simulated position are offset due to experimental delay.

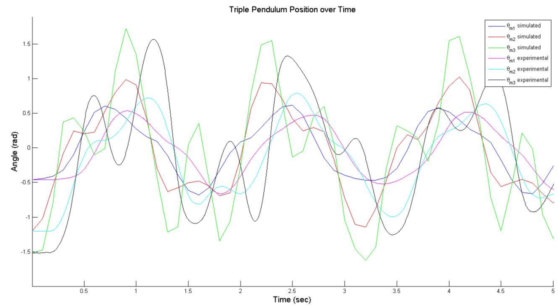

Fig. 5. The simulation and experimental data mostly match, save for the time delay offset

D. Velocity over time

The velocities of the third link have the higest fluctuation, due to its 3 degrees of freedom. Once again, the experimental and simulated position are offset due to experimental delay.

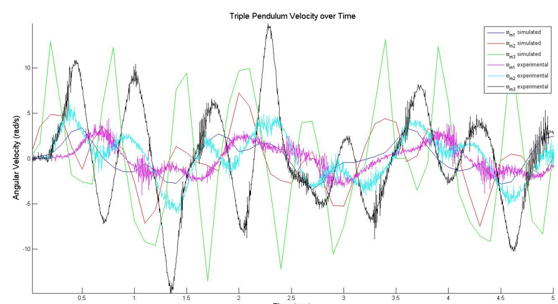

Fig. 6. Link 3 has the widest range of velocities

V. Animation

This is a snapshot of the animation as it runs. The animation displays the periodic movement of the pendulum.

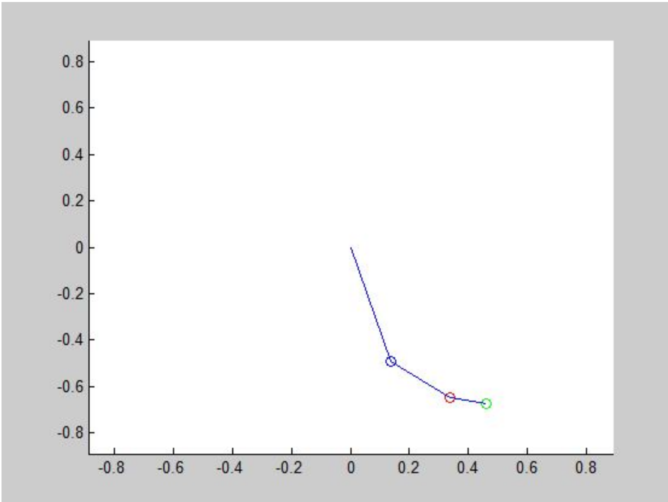

Fig. 7. Animation Capture

VI. Diagnosis

We originally tried to derive the equations of motion by hand, as we have done in the past. After several failed attempts to derive the correct equations, we decided to try using Mathematica. Although this involved the added task of learning how to use Mathematica software, it turned out to be a good decision. This allowed us to solve for long, complicated equations with minimal mistakes. However, processing the experimental data in MATLAB took more time than initally expected.

VII. Improvement

Most prominently, our model could be improved by adding viscous damping to the system. This would increase the accuracy of the simulation compared against the experiment. Furthermore, we could strengthen the model by no longer assuming that the masses have uniform density. This would complicate our moments of inertia and significantly complicate the model.

VIII. Reflection

Throughout this project, we learned how to use Lagrangian equations applied to a complex model. As opposed to our past experiements and analyses, we used Mathematica, a highly useful software, in order to solve for the equations of motion. We also improved at debugging MATLAB, diagnosing algebraic errors, and intuitively predicting how the model should behave.

IX. Conclusion

Because the initial positions of the pendulum's links were between 0 and 90 degrees, the behavior of the pendulum was fairly periodic and had no chaotic movements. Compared to the experimental data, the model closely matched the periodic movements of the pendulum, with the exception of a time offset. This delay was caused by holding the links at the start of the experiment, thus not releasing the pendulum immediately at time zero. Comparing our results with experimental data and with our past experiments, we were able to observe the accuaracy of modeling periodic systems and the difficulty in modeling a chaotic system.

X. Further Usage

Modeling a triple pendulum provides the opportunity to work through a complex system with a unique set of problems. The jump from from a double pendulum to a triple pendulum is significant, so much so that the algebra to derive the equations of motion necessitates a computer program. Also, a triple pendulum is a great vehicle to observe the behavior of chaotic systems.